MOMENTS

MATHEMATICAL EXPECTATION

The mean or Expectation of a random Variable is defined by

$$\bar{X} = E(X) = \begin{cases} \sum_i x_i P_i & \text{for discrete} \\ \int_{-\infty}^{\infty} x f(x)\,dx & \text{for Continuous} \end{cases}$$

Let

$$E(x^2) = \begin{cases} \sum_i x_i^2 P_i & \text{for discrete} \\ \int_{-\infty}^{\infty} x^2 f(x)\,dx & \text{for Continuous} \end{cases}$$

Note $V(x) = E(x^2) - (E(x))^2$

In general

$$E(x^n) = \begin{cases} \sum_i x_i^n P_i & \text{for discrete} \\ \int_{-\infty}^{\infty} x^n f(x)\,dx & \text{for Continuous} \end{cases}$$

Raw moment :

r^{th} moment about the origin of a RV X is defined as $\mu_r' = E(x^r) = \begin{cases} \sum_i x_i^r P_i & \text{for discrete} \\ \int_{-\infty}^{\infty} x^r f(x)\,dx & \text{for cont} \end{cases}$

Properties of Mean :

(i) $E(a) = a$ (ii) $E[ax \pm bY] = aE(x) \pm bE(Y)$

(iii) $E(x) \geq 0$ if $x \geq 0$

(iv) $E(XY) = E(X)E(Y)$ if X & Y are independent

(v) $P(X \geq a) \leq \frac{E(X)}{a}$, $a > 0$

(vi) $E(X+Y) = E(X) + E(Y)$

Properties of Variance :

(i) $V(a) = 0$ (ii) $V(ax) = a^2 V(x)$

(iii) $V(ax \pm b) = a^2 V(x)$

(iv) $V(ax \pm bY) = a^2 V(x) + b^2 V(Y)$

Moment Generating Function : (MGF)

$$M_x(t) = E(e^{tx}) = \begin{cases} \sum e^{tx} P(x) & \text{for discrete} \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx & \text{for continuous} \end{cases}$$

Note: $M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \mu_r'$

The coefficient of $\frac{t^r}{r!}$ in $M_x(t) = \mu_r'$

Note 2:

$$\left[\frac{d}{dt} M_x(t)\right]_{t=0} = \text{Mean} = E(x)$$

$$\left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0} = E(x^2)$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

CENTRAL MOMENT:

r^{th} moment about the mean is

$$\mu_r = E(x-\bar{x})^r = \begin{cases} \sum (x-\bar{x})^r p_i & \text{for discrete} \\ \int_{-\infty}^{\infty} (x-\bar{x})^r f(x)\,dx & \text{for continuous} \end{cases}$$

Relationship between Raw moments and Central Moments:

(i) $\mu_1 = 0$

(ii) $\mu_2 = \mu_2' - \mu_1'^2$

7 (iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

(iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$